AFCRC-TN-58-130
Astia Document No. AD 146887

NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

RESEARCH REPORT No. EM-110

The Inverse Scattering Problem in Geometrical Optics and the Design of Reflectors

JOSEPH B. KELLER

Contract No. AF 19(604)1717

January, 1958

NEW YORK UNIVERSITY

Institute of Mathematical Sciences

Division of Electromagnetic Research

Research Report No. EM-110

THE INVERSE SCATTERING PROBLEM IN GEOMETRICAL OPTICS AND THE DESIGN OF REFLECTORS

Joseph B. Keller

Joseph B. Keller

Joseph B. Keller

Morris Kline

Morris Kline
Project Director

The research reported in this document has been sponsored by the Air Force Cambridge Research Center, Air Research and Development Command, under Contract No. AF19(604)1717.

New York, 1958

Abstract

The inverse scattering problem considered here is that of finding the shape of a reflector which produces a prescribed scattered wave. The scattered wave is characterized by its angular pattern, which determines the differential scattering cross section of the reflector. The problem is solved by means of explicit formulas for cylindrical and for rotationally symmetric objects. Plane, cylindrical and spherical incident waves are considered. The general three dimensional object is also treated. The method of geometrical optics is used throughout.

Table of Contents

1. Introduction

When radiation of any type is incident upon an object some of the radiation is scattered in all directions by the object. The direct problem of the theory of scattering is that of determining the intensity of the radiation scattered in each direction when the properties of the incident radiation as well as those of the object are known. The inverse scattering problem is that of determining the properties of the scattering object when the incident radiation and the intensity of the radiation scattered in each direction are known. This latter problem has been studied extensively in atomic and nuclear physics by the methods of quantum mechanics and, to some extent, by classical mechanics [1]. The classical mechanical solution is also applicable to the scattering of light (or other radiation obeying the laws of geometrical optics) by a region in which the index of refraction varies in any continuous spherically symmetric manner. We now propose to consider the inverse problem in geometrical optics when the scatterer is an opaque object, such as a piece of metal, with a definite boundary. We will call an object of this type a reflector or mirror.

In the two dimensional case, which we treat first, the scattered wave is cylindrical. Its intensity in the direction θ is denoted by $\sigma(\theta)$. When the incident radiation is a plane wave of unit amplitude, $\sigma(\theta)$ is called the differential scattering cross section of the reflector. We then find that the shape of the reflector is completely determined if $\sigma(\theta)$ and the reflection coefficient of the reflector are given. However the reflector may be placed so that either its concave or convex side is exposed to the incident radiation. When the incident radiation is a cylindrical wave a one parameter family of different reflectors is found. In both cases formulas for the shape of the reflector are obtained in terms of $\sigma(\theta)$. Similar results are obtained for reflectors which are surfaces of revolution in

three dimensions, when the incident radiation is either a plane wave incident along the axis of revolution or a point source on this axis.

In the general three dimensional case the differential scattering cross section $\sigma(\theta,\phi)$ and the reflection coefficient of the surface do not determine the shape of the reflector at all. However, if the differential scattering cross section is known for two plane waves incident from opposite directions, and if the reflection coefficient is also known, then the reflector is uniquely determined, provided that it is convex. But in this case it has not been possible to determine the shape explicitly. Instead a non-linear partial differential equation must be solved to find the shape.

Our investigation was pursued not only because of its intrinsic interest, but also because the results may be of use in the design of optical systems containing reflectors. This, as well as most design problems, may profitably be viewed as an inverse problem. For purposes of comparison we have also included some well-known results on the direct problem.

S. N. Karp has also treated certain inverse scattering problems for reflectors without restriction to the realm in which geometrical optics is valid.

2. Formulation and solution

In geometrical optics light propagates along rays which, in a homogeneous medium, are straight lines. Therefore by conservation of energy, in a non-absorbing medium, the flux of energy is the same through every cross section of each tube of rays. This flux is proportional to the light intensity I multiplied by the cross sectional area of the tube. In a cylindrical wave a tube of angular width $d\theta$ has a cross section of length $rd\theta$ so energy conservation yields $Ird\theta = \sigma\, d\theta$ where σ is constant along the tube. Thus $I = \frac{\sigma(\theta)}{r}$. Here σ is written as $\sigma(\theta)$ since it may vary from one tube to another. Now suppose the cylindrical wave is produced by

reflection from a reflector of an incident plane wave of unit intensity propagating from the left parallel to the x-axis. (See Fig. 1) Suppose the ray at height y is reflected in the direction θ, and that at height $y + dy$ is reflected in the direction $\theta + d\theta$. Let the energy reflection coefficient be R. Then of the incident energy between the two rays, which is proportional to dy, the fraction Rdy is reflected into the tube between the rays in the directions θ and $\theta + d\theta$. Thus by the conservation of energy

$$(1) \qquad R\,dy = \pm\,\sigma(\theta)\,d\theta\,.$$

The sign in (1) must be chosen the same as that of $dy/d\theta$.

The reflection coefficient R may not be a constant, but may be a function of the angle of incidence. This angle is just $\theta/2$ since the normal to the reflector at each point bisects the angle between the incident and reflected rays at this point. Thus in (1), $R = R(\theta/2)$. If y and x are coördinates of a point on the reflector surface then the slope of the normal also leads to the equation

$$(2) \qquad \frac{dy}{dx} = \cot\frac{\theta}{2}\,.$$

Equations (1) and (2) enable us to solve the inverse problem of finding the reflector shape in terms of $\sigma(\theta)$ and $R(\theta/2)$. For integrating (1) yields

$$(3) \qquad y = y_0 \pm \int_0^\theta \frac{\sigma(\theta)}{R(\theta/2)}\,d\theta.$$

Elimination of dy from (1) and (2) and integration of the result also gives

$$(4) \qquad x = x_0 \pm \int_0^\theta \frac{\sigma(\theta)\tan\frac{\theta}{2}}{R(\theta/2)}\,d\theta.$$

Equations (3) and (4) are parametric equations for the reflector and thus provide the solution of the inverse problem. The integration constants x_0 and y_0 determine the location of the reflector. The sign, which must be the same in both equations

determines whether the reflector is convex (+) or concave (-) toward the incident wave. In the concave case the specified $\sigma(\theta)$ is obtained only for a limited range of θ due to blocking of the reflected rays by the reflector itself.

As an example of the use of (3) and (4), let us suppose that

$$\sigma(\theta) = \frac{b}{2} R\left(\frac{\theta}{2}\right) \cos\frac{\theta}{2} . \tag{5}$$

Here b is a constant. Now (3) and (4) yield

$$x = x_0 \pm b\left[1 - \cos\frac{\theta}{2}\right] \tag{6}$$

$$y = y_0 \pm b \sin\frac{\theta}{2} . \tag{7}$$

These are the equations of a circle of radius b centered at $x_0 \pm b$, y_0. (See Fig. 2)

As a second example let a be a constant and suppose that

$$\sigma(\theta) = \frac{aR}{2} . \tag{8}$$

If R is constant then σ is independent of θ and the corresponding reflector is an isotropic scatterer. From (3) and (4) its equations are

$$x = x_0 \mp a \log \cos\frac{\theta}{2} \tag{9}$$

$$y = y_0 \pm \frac{a\theta}{2} . \tag{10}$$

In Fig. 3 this reflector is drawn, using (9) and (10) with the upper sign.

Equation (1) can also be used to solve the direct problem of scattering. If $b(\theta/2)$ denotes the radius of curvature of the reflector at the point where its normal points in the direction $\theta/2$ then simple geometry shows that

$$\frac{dy}{d\theta} = \frac{1}{2} b(\theta/2) \cos\frac{\theta}{2} . \tag{11}$$

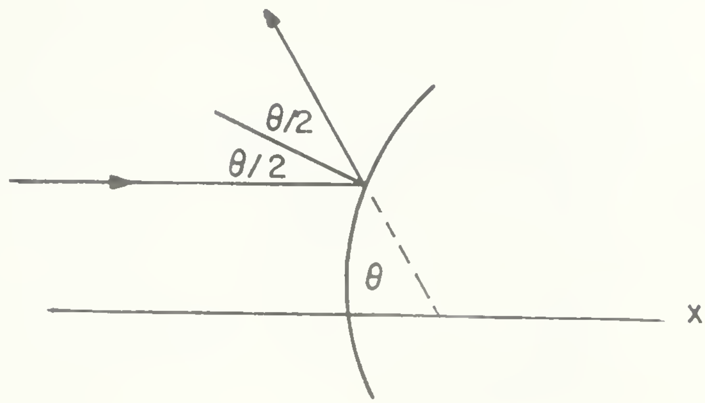

Fig. 1: Incident and reflected rays both making the angle $\theta/2$ with the normal to the reflector. The reflected ray makes the angle θ with the negative x-axis, along which the incident rays are incident. If the reflector is a surface of revolution, the figure represents a cross section containing the axis of revolution.

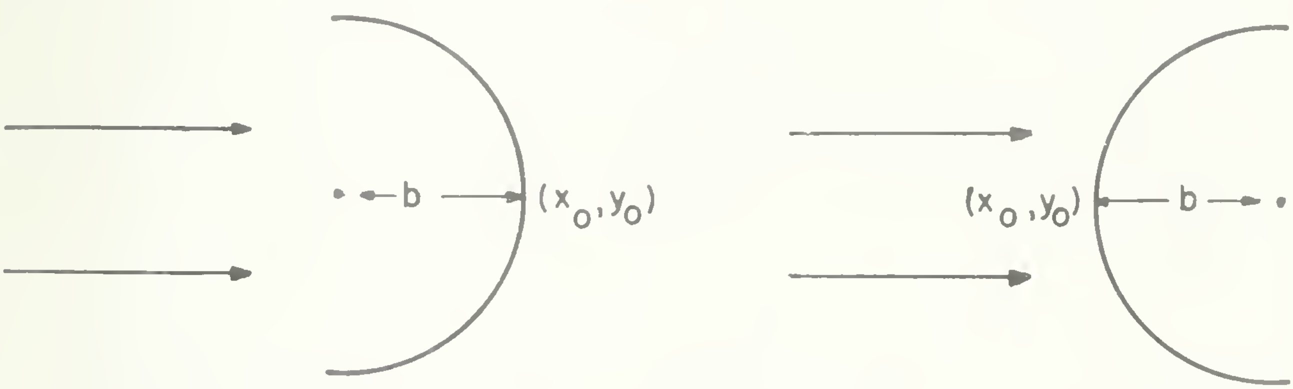

Fig. 2: A semicircular reflector of radius b which is either concave (A) or convex (B) to the incident wave. In both cases the differential scattering cross section is the same, being given by (5).

When (11) is used in (1), the result is

$$\sigma(\theta) = \frac{1}{2} R(\tfrac{\theta}{2})\, b(\tfrac{\theta}{2}) \cos \frac{\theta}{2} \quad . \tag{12}$$

This is the solution of the direct problem. It can be used in solving the inverse problem, but the method used above is simpler.

3. Other cases

By exactly the same methods as those used above we have also considered the direct and inverse problems for three other cases. These are the two dimensional case with an incident field due to a line source and the three dimensional case of a reflecting surface of revolution with a plane wave incident along the axis of revolution or with a point source located on this axis. The notation in the plane wave case is the same as in Fig. 1 except that r is used instead of y and z instead of x. For the line and point source cases the notation is explained in Fig. 4. In these cases ϕ denotes the angle of incidence and of reflection and α denotes the angle between the incident ray and the axis. These angles are related to θ by

$$\theta = 2\phi - \alpha. \tag{13}$$

The intensity on each incident ray is denoted by $I(\alpha)$ and the equation of the reflector is written as $\rho = \rho(\alpha)$ where the source is the origin. The results are given in the Table.

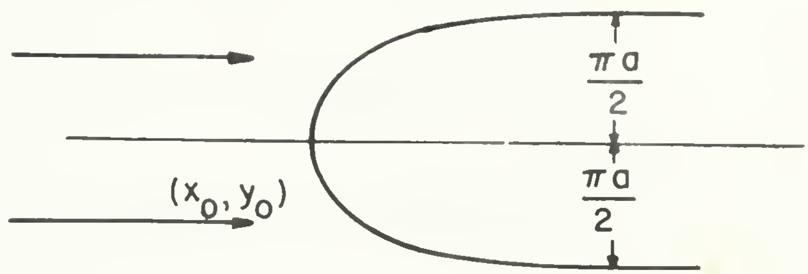

Fig. 3: An isotropic scatterer given by (9) and (10) with the upper sign. If the reflection coefficient R is constant, this reflector has the constant differential cross section $\sigma = aR/2$. Its width is asymptotically πa.

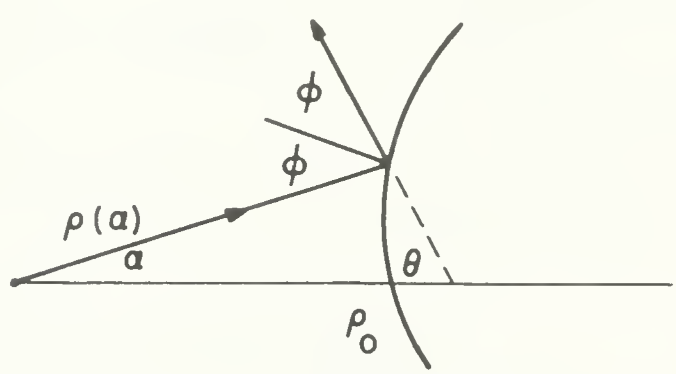

Fig. 4: An incident ray making the angle α with the x or z axis and the angle ϕ with the normal to the reflector. The reflected ray also makes the angle ϕ with the normal and the angle θ with the axis. The distance from the source to the reflector is $\rho(\alpha)$, and $\rho(0)$ is also denoted by ρ_0.

Results for Inverse Scattering

Two dimensions, line source

$$\pm\, \sigma(\theta)\, d\theta \;=\; R(\phi)\, I(\alpha)\, d\alpha$$

$$\rho^{-1} \frac{d\rho}{d\alpha} \;=\; \tan \frac{\alpha+\theta}{2}$$

If R = constant

$$\pm \int_{\theta_o}^{\theta} \sigma(\theta)\, d\theta = R \int_{o}^{\alpha} I(\alpha) d\alpha$$

$$\log \rho/\rho_o \;=\; \int_{o}^{\alpha} \tan \frac{\alpha+\theta(\alpha)}{2}\, d\alpha \, .$$

Rotational symmetry, plane wave

$$\sigma(\theta) \sin\theta\, d\theta \;=\; R(\tfrac{\theta}{2})\, r\, dr$$

$$\frac{dr}{dz} \;=\; \pm \cot \frac{\theta}{2}$$

If R = constant

$$r^2 \;=\; 2 \int_{0}^{\theta} \frac{\sigma(\theta) \sin\theta}{R(\theta/2)}\, d\theta$$

$$z \;=\; z_o \pm \int_{0}^{\theta} \frac{\sigma(\theta) \sin\theta \tan\frac{\theta}{2}}{R(\theta/2) r(\theta)}$$

Rotational symmetry, point source

$$\pm\, \sigma(\theta) \sin\theta\, d\theta \;=\; R(\phi) I(\alpha) \sin\alpha\, d\alpha$$

$$\rho^{-1} \frac{d\rho}{d\alpha} \;=\; \tan \frac{\alpha + \theta(\alpha)}{2}$$

If R = constant

$$\pm \int_{\theta_o}^{\theta} \sigma(\theta) \sin\theta\, d\theta = R \int_{0}^{\alpha} I(\alpha) \sin\alpha\, d\alpha$$

$$\log \rho/\rho_o \;=\; \int_{0}^{\alpha} \tan \frac{\alpha + \theta(\alpha)}{2}\, d\alpha$$

Results for Direct Scattering

Two dimensions, line source

$$\sigma(\theta) = R(\phi)\, I(\alpha) \left[\frac{2\rho(\alpha)}{b(\alpha)\cos\phi} + 1 \right]^{-1}$$

Rotational symmetry, plane wave

$$\sigma(\theta) = \frac{r(\theta/2)}{2\sin\theta}\, R(\theta/2)\, b(\theta/2) \cos\theta/2$$

Rotational symmetry, point source

$$\sigma(\theta) = \frac{\sin\alpha}{2\sin\theta}\, R(\phi)\, I(\alpha) \left[\frac{2\rho(\alpha)}{b(\alpha)\cos\phi} + 1 \right]^{-1} .$$

As an example let us consider the two dimensional case in which the field comes from a line source. Suppose that σ, I and R are constants and that $\theta_o = 0$. Then from the table we have

(14) $$\pm\,\sigma\theta = IR\alpha$$

(15) $$\rho = \rho_o \left[\cos\left(1 \pm \frac{IR}{\sigma}\right) \frac{\alpha}{2} \right]^{-2\left(1 \pm (IR/\sigma)\right)^{-1}} .$$

Equation (15) is the polar equation of the reflector. If the positive sign is taken and if $IR = \sigma$ then it is the equation of a plane at distance ρ_o from the source.

In the three dimensional case with rotational symmetry and a plane wave incident along the axis, let us seek a reflector for which σ and R are constants. From the table we find for its equation

(16) $$r^2 = \frac{4\sigma}{R} \sin^2 \frac{\theta}{2}$$

(17) $$z = z_0 \pm 2\sqrt{\sigma/R} \left(1 - \cos \frac{\theta}{2}\right)$$

This is the equation of a sphere of radius $2\sqrt{\sigma/R}$ centered at $r = 0$, $z = z_0 \pm 2\sqrt{\sigma/R}$. Thus we find the well-known result that the sphere is an isotropic scatterer - in fact the only one, in this case.

4. The general case

Suppose the reflector is an arbitrary smooth convex closed surface in three dimensional space. Let $\sigma(\theta,\phi)$ denote its differential scattering cross section in the direction θ, ϕ due to a plane wave of unit intensity incident from the left along the x-axis. Let $P(\theta,\phi)$ be the point on the reflector from which a ray is reflected in the θ, ϕ direction. Then the normal at $P(\theta,\phi)$ must point in the direction $\theta/2$, ϕ and the angle of incidence at P must be $\theta/2$. These facts follow from the law of reflection. Now it is well known that the differential scattering cross section in the direction of any reflected ray is equal to $R/4G$ where G is the Gaussian curvature at the point of reflection. Therefore we have

(18) $$\sigma(\theta,\phi) = \frac{R(\frac{\theta}{2})}{4G(\frac{\theta}{2}, \phi)} \quad .$$

Let us make use of (18) to analyze the inverse problem. We see that when σ and R are known for all values of their arguments, the Gaussian curvature G is determined over the hemisphere $0 \leq \theta < \frac{\pi}{2}$ of the unit sphere. This unit sphere, each point of which corresponds to the direction of the normal at one point on the reflector surface, is called the spherical image of that surface. The problem of determining a surface when its Gaussian curvature is given on the entire

surface of the spherical image is known as Minkowski's problem. [3] It has one and only one solution for any sufficiently smooth positive function $G(\theta,\phi)$ which satisfies the condition

$$(19) \qquad \int G^{-1}(\theta,\phi)\, \vec{n}\,(\theta,\phi)\, d\Omega = 0 \quad .$$

Here $\vec{n}(\theta,\phi)$ is the unit normal at the point θ, ϕ; $d\Omega$ is the element of area of the unit sphere and the integration extends over the whole sphere. Thus for any arbitrary (subject to (19)) smooth continuation over the rest of the sphere of the function G given on half the sphere by (18), there is exactly one reflector. The reflector shape is determined by the solution of an elliptic partial differential equation involving G. Therefore we may conclude that the shape of all parts of the reflector are effected by the arbitrary continuation of G. Consequently the data provided by (18) do not suffice to determine the shape of the reflector nor any part of it. The inverse problem has too large a family of solutions.

Suppose, however, that two functions $\sigma_+(\theta,\phi)$ and $\sigma_-(\theta,\phi)$ are given, corresponding to two different incident waves coming from opposite directions. If R is also known, then (18) determines G over the whole sphere. If this G satisfies (19) - as it must if it actually corresponds to a surface - then the inverse problem has a unique solution. The calculation of this solution, in the general case, requires the solution of an elliptic partial differential equation. In the cases treated in the previous sections this equation could be reduced to an ordinary differential equation. This explains why explicit solutions could be obtained in those cases.

References

[1] J.B. Keller, I. Kay and J. Shmoys - Phys. Rev.; 102, No. 2, April 15, 1956, pp. 557-559.

The procedure of [1] requires the determination of $\theta(b)$ from $\sigma(\theta)$, which is possible if and only if at most one trajectory is scattered in each direction. Then $V(r)$ is found from $\theta(b)$ provided that $[1 - E^{-1}V(r)]r^2$ is a monotone increasing function of r for $r \geq r_{min}(E)$. Monotonicity is required to permit the change of integration variable from u in (3) to w in (5). It also ensures that there are no bounded orbits in the region $r \geq r_{min}(E)$, while without monotonicity there are such orbits. Thus $\theta(b)$ at energy E determines $V(r)$ for $r \geq r_{min}(E)$ provided that there are no bound states at energy E in the region $r \geq r_{min}(E)$. If there is such a bound state the method of [1] does not work, and it seems that it can be modified to show that $V(r)$ is then not uniquely determined by $\theta(b)$.

[2] S. Silver, Ed. - Microwave Antenna Theory and Design (Rad. Lab. Series No. 12) McGraw-Hill, N.Y. (1949) pp. 497-500.

In this reference it is not made clear that the constant K is actually the reflection coefficient R.

[3] L. Nirenberg - Comm. Pure and Appl. Math.; VI, No. 3, August 1953, pp. 337-394.

DISTRIBUTION LIST FOR RESEARCH REPORTS

Contract No. AF 19(604)1717

(ONE copy unless otherwise noted)

Commander
Air Force Missile Test Center
Patrick Air Force Base, Florida
ATTN: Technical Library

Commander
Air Research and Development Command
Washington 25, D.C.
ATTN: RDDL

Commander
Air Research and Development Command
Washington 25, D.C.
ATTN: Major E. Wright, RDTDRC-1

Director
Air University Library
Maxwell Air Force Base, Alabama
ATTN: CR-4582

Commander
Air Force Missile Test Center
Patrick Air Force Base, Florida
ATTN: Technical Library

Headquarters, USAF
Director of Research and Development
Washington 25, D.C.
ATTN: AFDRD

Commander
Wright Air Development Center
Aright-Patterson Air Force Base, Ohio
ATTN: Mr. Walter J. Portune, WCLRS-6

Commander
Wright Air Development Center
Wright-Patterson Air Force Base, Ohio
ATTN: WCLNT-5, N. Draganjac

Commander
Wright Air Development Center
Wright-Patterson Air Force Base, Ohio
ATTN: WCLRE-5, Wave Propagation Section

Air Technical Intelligence Center
Wright-Patterson Air Force Base, Ohio
ATTN: AFOIN-4B1a

(2)Commander
Rome Air Development Center
Griffiss Air Force Base, New York
ATTN: RCSST-3

Commander
Rome Air Development Center
Griffiss Air Force Base, New York
ATTN : D. Mather, RCERT

Rome Air Development Center
Griffiss Air Force Base, New York
ATTN: Dr. John Burgess, RCER

Commander
Holloman Air Research and Development Center
Holloman Air Force Base, New Mexico
ATTN: HDOI

Director
Ballistics Research Laboratory
Aberdeen Proving Ground, Maryland
ATTN: Ballistics Measurement Laboratory

Ballistic Research Laboratories
Aberdeen Proving Ground, Maryland
ATTN: Technical Information Branch
Victor W. Richard

Director
Evans Signal Laboratory
Belmar, New Jersey
ATTN: Mr. O.C. Woodyard

Director Communications and Electronics
Hq., U.S. Air Force
Washington 25, D.C.
ATTN: AFDAC s/e

Director
Evans Signal Laboratory
Belmar, New Jersey
ATTN: Technical Document Center

Massachusetts Institute of Technology
Signal Corps Liaison Officer
Cambridge 39, Massachusetts
ATTN: A.D. Bedrosian, Room 20C-116

Commanding General
SC Engineering Laboratories
Fort Monmouth, New Jersey
ATTN: Sceltechnical Documents Center
Bay 2C122

Department of the Army
Office of Chief Signal Officer
Washington 25, D.C.
ATTN: SIGGD

Department of the Army
Office of Chief Signal Officer
Engineering and Technical Division
Washington 25, D.C.
ATTN: SIGNET-5

Diamond Ordnance Fuze Laboratories
Washington 25, D.C.
ATTN: Guided Missile Fuze Library
R.D. Hathcher,
Chief Microwave Dev. Section

(10)Armed Services Tech. Information Agency
Arlington Hall Station
Arlington 12, Virginia
ATTN: DSC-SA

National Bureau of Standards
Boulder Laboratories Library
Boulder, Colorado
ATTN: Mrs. Victorie S. Barker, Lib.

National Bureau of Standards
Department of Commerce
Washington 25, D.C.
ATTN: Dr. A.G. McNish

National Bureau of Standards
Department of Commerce
Washington 25, D.C.
ATTN: Gustave Shapiro
Electronic Miniaturization Lab.
Engineering Electronics Section
Electricity and Electronics Div.

Director
National Bureau of Standards
Central Radio Propagation Lab.
Boulder, Colorado
ATTN: Division 14.0

(2)Office of Technical Services
Department of Commerce
Washington 25, D.C.
ATTN: Technical Reports Section

Library, Div. of Public Documents
U.S. Government Printing Office
Washington 25, D.C.

Director, National Security Agency
Washington 25, D.C.
ATTN: R/D (331)

(2)Hq. Air Force Cambridge Research Center
Air Research and Development Command
Laurence G. Hanscom Field
Bedford, Mass.
ATTN: CROOTR-2 - P. Condon

(5)Hq. Air Force Cambridge Research Center
Air Research and Development Command
Laurence G. Hanscom Field
Bedford, Mass.
ATTN: CROOTR - J. Armstrong

(3)Hq. Air Force Cambridge Research Center
Air Research and Development Command
Laurence G. Hanscom Field
Bedford, Mass.
ATTN: CRRD - R.E. Hiatt

Chief, Bureau of Aeronautics
Department of the Navy
Washington 25, D.C.
ATTN: Code Aer-TD-414

Chief, Bureau of Ships
Navy Department
Washington 25, D.C.
ATTN: Mr. E. Johnston, Code 838

Commander
U.S. Naval Air Missile Test Center
Point Mugu, California
ATTN: Code 366, MT-6

U.S. Naval Ordnance Laboratory
White Oak, Silver Spring 19, Maryland
ATTN: The Library

Commander
U.S. Naval Ordnance Test Station(Code 753)
China Lake, California
ATTN: Tech. Library Branch

Librarian
U.S. Naval Postgraduate School
Monterey, California

Director
U.S. Naval Research Laboratory
Washington 25, D.C.
ATTN: Code 2021

Director
U.S. Naval Research Laboratory
Washington 25, D.C.
ATTN: Code 1110

Director
U.S. Naval Research Laboratory
Washington 25, D.C.
ATTN: J. Bohnert, Code 5250

Commander
U.S. Navy Underwater Sound Laboratory
Fort Trumbull, New London Conn.
ATTN: Mr. C.M. Dunn

Chief of Naval Research
Navy Department, Washington 25, D.C.
ATTN: Code 427

(2)Commanding Officer and Director
U.S. Navy Electronics Lab. Library
San Diego 52, California

Chief, Bureau of Ordnance
Navy Department, Washington 25, D.C.
ATTN: Technical Library, Code Ad3
Mr. H. B. Rex, Code ReS4

Chief, Bureau of Ordnance
Navy Department, Washington 25, D.C.
ATTN: Mr. C.H. Jackson, Code Re9a

Chief, Bureau of Ordnance
Navy Department, Washington 25, D.C.
ATTN: L.E. Wallingford, Code Re4F

Chief, Bureau of Aeronautics
Department of the Navy, Washington 25, D.C.
ATTN: Code Aer-EL-931

(2)Commander
U.S. Naval Air Development Center
Johnsville, Pennsylvania

Commander
U.S. Naval Air Test Center
Patuxent River, Maryland
ATTN: ET-315, Antenna Section

Naval Ordnance Laboratory
Corona, California
ATTN: Library

U.S. Naval Ordnance Laboratory
Dept. of the Navy
Corona, California
ATTN: Dr. Robert L. Conger

Airborne Instruments Laboratory, Inc.
160 Old Country Road
Mineola, New York
Attn: Dr. E.G. Fubini, Director
Research and Engineering Division

Aircom, Inc.
354 Main Street
Winthrop, Mass.

American Machine and Foundry Company
Electronics Division
1085 Commonwealth Avenue
Boston 15, Mass.
Attn: Mrs. Rita Moravcsik, Librarian

Andrew Alford Consulting Engineers
299 Atlantic Avenue
Boston 10, Mass.

Avion Division
ACF Industries, Inc.
800 North Pitt Street
Alexandria, Virginia
Attn: Technical Library

Battelle Memorial Institute
505 King Avenue
Columbus 1, Ohio
Attn: Dr. Bernard H. List

Bell Aircraft Corporation
P.O. Box One, Buffalo 5, N.Y.
Attn: Mrs. J. Mulcahey, Tech. Lib.

Bell Telephone Laboratories, Inc.
Whippany Laboratory, Whippany, N.J.
Attn: Technical Information Lib.

Bendix Aviation Corporation
Pacific Division, 11600 Sherman Way
North Hollywood, Calif.
Attn: J.R. Breninger, Engr. Librarian

Bendix Radio, Div. of Bendix Aviation Corp.
East Joppa Road, Towson 4, Maryland
Attn: Dr. D.M. Allison, Jr.
Dir. Engineering and Research

Bjorksten Research Laboratories, Inc.
P.O. Box 265
Madison 1, Wisconsin
Attn: F.B. Korsgard, Treasurer

Boeing Airplane Company
Pilotless Aircraft Division
Seattle 24, Washington
Attn: R.R. Barber, Library Supervisor

Boeing Airplane Company
Wichita Division Engineering Library
Wichita 1, Kansas
Attn: Kenneth C. Knight, Librarian

Boeing Airplane Company
Seattle Division
Seattle 14, Washington
Attn: E.T. Allen, Library Supervisor

Convair, A Division of General Dynamics Corp.
Fort Worth, Texas
Attn: K.G. Brown, Division Research Lib.

Convair, A Div. of General Dynamics Corp.
San Diego 12, California
Attn: Mrs. Dora B. Burke, Engr. Lib.

Cornell Aero. Laboratory, Inc.
4455 Genesee Street
Buffalo 21, New York
Attn: Elma T. Evans, Librarian

Dalmo Victor Company
A Division of Textron, Inc.
1515 Industrial Way
Belmont, California
Attn: Mary Ellen Addems, Tech. Lib.

Dorne and Margolin, Inc.
30 Sylvester Street
Westbury, Long Island, N.Y.

Douglas Aircraft Company, Inc.
P.O. Box 200
Long Beach 1, California
Attn: Engineering Lib. (C-250)

Douglas Aircraft Company, Inc.
827 Lapham Street
El Segundo, California
Attn: Engineering Library

Douglas Aircraft Company, Inc.
3000 Ocean Park Boulevard
Santa Monica, California
Attn: Eq. Sec. Ref. Files, Eq. Eng. A250

Douglas Aircraft Corporation
2000 North Memorial Drive
Tulsa, Oklahoma
Attn: Engineering Library, D-250

Electronic Communication, Inc.
St. Paul Street at University Parkway
The Marylander Apartments
Baltimore 18, Maryland
Attn: Dr. D.D. King, Vice-President

Electronic Defense Laboratory
P.O. Box 205
Mountain View, California
Attn: Library

Electronics Research, Inc.
2300 N.New York Avenue
Evansville, Indiana
Attn: B.H. Baldridge, Engr. Supervisor

Emerson and Cuming, Inc.
869 Washington Street
Canton, Mass.
Attn: W.R. Cuming

The Emerson Electric Mfg. Company
8100 Florissant Avenue
St. Louis 21, Missouri
Attn: Mr. E. Breslin, Librarian

Fairchild Aircraft
Division Fairchild Eng. and Airplane Corp.
Hagerstown, Maryland
Attn: Library

Farnsworth Electronics Company
Fort Wayne 1, Indiana
Attn: Technical Library

Federal Telecommunication Laboratories
500 Washington Avenue
Nutley 10, New Jersey
Attn: Technical Library

Gabriel Electronics
Division of Gabriel Company
135 Crescent Road
Needham Heights 94, Mass.
Attn: Mr. Steven Galagan

General Electric Company
Ithaca, New York
Attn: J.B. Travis,
Advanced Electronics Center

General Electric Company
Electronics Park
Syracuse, New York
Attn: Bessie Fletcher, Documents Library
Bldg. 3-143A

General Precision Laboratory, Inc.
63 Bedford Road
Pleasantville, New York
Attn: Mrs. Mary G. Herbst, Librarian

Goodyear Aircraft Corp.
1210 Massillon Road
Akron 15, Ohio
Attn: Library D/120 Plant A

Granger Associates
Electronic Systems
966 Commercial Street
Palo Alto, California
Attn: John V.N. Granger, Pres.

Grumman Aircraft Engineering Corp.
Bethpage, Long Island, N.Y.
Attn: Mrs. A.M. Gray, Librarian
Engineering Library, Plant No. 5

The Hallicrafters Co.
4401 West 5th Avenue
Chicago 24, Illinois
ATTN: L. La Gioia, Librarian

Hoffman Laboratories, Inc.
3740 South Grand Avenue
Los Angeles 7, California
Attn: Engineering Library

Hughes Aircraft Co., Antenna Dept.
Microwave Laboratory, Bldg. 12, Rm. 2617
Culver City, California
Attn: M.D. Adcock

Hughes Aircraft Co.
Florence and Teale Streets
Culver City, California
Attn: Dr. L.C. Van Atta, Assoc. Dir.
Research Laboratories

Hycon Eastern, Inc.
75 Cambridge Parkway
Cambridge, Mass.
Attn: Mrs. Lois Seulowitz, Tech. Lib.

International Business Machines Corp.
Military Products Division
590 Madison Avenue
New York 22, New York
Attn: Mr. C.F. McElwain, General Manager

International Business Machines Co.
Military Products Div.
Oswego, New York
Atnn: Mr. D.I. Marr, Librarian
Dept. 459

International Resistance Company
401 N. Broad Street
Philadelphia 8, Pa.
Attn: Research Library

Jansky and Bailey, Inc.
1339 Wisconsin Avenue, N.W.
Washington 7, D.C.
Attn: Mr. Delmer C. Ports

Dr. Henry Jasik, Consulting Engr.
298 Shames Drive,
Brush Hollow Industrial Park
Westbury, New York

Martin Katzin and Company, Consultants
711 14th Street, N.W.
Washington 5, D.C.

Lockheed Aircraft Corp.
Missile Systems Division
7701 Woodley Avenue
Van Nuys, California
Attn: Van Nuys Lib. Bldg. 902

Lockheed Aircraft Corp.
California Division Engr. Lib.
Dept. 72-25, Plant A-1, Bldg. 63-1
Burbank, California
Attn: N.C. Harnois

The Glenn L. Martin Company
Denver Division
Waterton, Colorado
Attn: William V. Foley,
Test and Reliability
Mail No. 2-38

The Glenn L. Martin Company
Baltimore 3, Maryland
Attn: Engr. Lib.. Antenna Design Group

Maryland Electronic Mfg. Corp.
5009 Calvert Road
College Park, Maryland
Attn: Mr. H. Warren Cooper

Mathematical Reviews
190 Hope Street
Providence 6, Rhode Island

The W.L. Maxson Corporation
460 West 34th Street
New York, N.Y.
Attn: Miss Dorothy Clark

McDonnell Aircraft Corp.
Lambert Saint-Louis Municipal Airport
Box 516, St. Louis 3, Missouri
Attn: R.D. Detrich, Engr. Library

McMillan Laboratory, Inc.
Brownville Avenue
Ipswich, Massachusetts
Attn: Security Officer, Document Rm.

Melpar, Inc.
3000 Arlington Blvd.
Falls Church, Va.
Attn: Engineering Tech. Lib.

Microwave Development Laboratory
90 Broad Street
Babson Park 57, Mass.
Attn: N. Tucker, General Manager

Microwave Radiation Company
19223 South Hamilton St.
Gardena, California
Attn: Mr. Morris J. Ehrlich, Pres.

Naval Industrial Reserve Aircraft Plant
P.O. Box 5907
Dallas, Texas
Attn: Mr. H.S. White, Librarian

Northrop Aircraft, Inc.
Hawthorne, California
Attn: Mr. E.A. Freitas, Lib. Dept. 3145

North American Aviation, Inc.
12214 Lakewood Blvd.
Downey, California
Attn: Engr. Lib. 495-115

North American Aviation, Inc.
Los Angeles International Airport
Los Angeles 45, California
Attn: Engineering Technical File

Page Communications Engineers, Inc.
710 Fourteenth Street, Northwest
Washington 5, D.C.
Attn: Librarian

Philco Corporation Res. Division
Branch Library
4700 Wiesachickon Avenue
Philadelphia 44, Pa.
Attn: Mrs. Dorothy S. Collins

Pickard and Burns, Inc.
240 Highland Avenue
Needham 94, Mass.
Attn: Dr. J.T. deBettencourt

Polytechnic Research and Dev. Co., Inc.
202 Tillary Street
Brooklyn 1, New York
Attn: Technical Library

Radiation Engineering Laboratory
Main Street
Maynard, Mass.
Attn: Dr. John Ruze

Radiation, Inc.
P.O. Drawer 37
Melbourne, Florida
Attn: Tech. Library, Mr. M.L. Cox,
Assistant Project Engineer

Radio Corp. of America
RCA Laboratories
Rocky Point, New York
Attn: P.S. Carter, Lab. Library

RCA Laboratories
David Sarnoff Research Center
Princeton, New Jersey
Attn: Miss Fern Cloak, Lib., Res. Library

RCA Defense Electronic Products
Camden 2, New Jersey
Attn: Clarence A. Gunther, Chief Def. Engr.
DEP - Bldg. 15, Floor 7

(2)The Ramo-Wooldridge Corporation
5730 Arbor Vitae Street
Los Angeles 45, California
Attn: Margaret C. Whitnah, Chief Lib.

Hoover Microwaves
9592 Baltimore Avenue
College Park, Maryland

The Rand Corporation
1700 Main Street
Santa Monica, California
Attn: Library - via: SBAMA Liaison Office

Rantec Corporation
Calabasas, California
Attn: Grace Keener, Office Manager

Raytheon Manufacturing Company
Missile Systems Division
Bedford, Mass.
Attn: Mr. Irving Goldstein

Raytheon Manufacturing Company
Wayland Laboratory
Wayland, Mass.
Attn: J.E. Walsh, Dept. 8974,
Antenna Design Radar

Raytheon Manufacturing Company
Wayland Laboratory
Wayland, Mass.
Attn: Miss Alice G. Anderson, Librarian

Republic Aviation Corporation
Farmingdale, Long Island, N.Y.
Attn: R.E. Fidoten, Engineering Lib.

Rheem Manufacturing Company
Research and Development
9236 East Hall Road
Downey, California
Attn: J.C. Joerger

Ryan Aeronautical Company
Lindbergh Field
San Diego 12, California
Attn: Library

Sage Laboratories, Inc.
30 Guinan Street
Waltham 54, Mass.

Sanders Associates, Inc.
95 Canal Street
Nashua, New Hampshire
Attn: N.R. Wild, Library

Sandia Corporation, Sandia Base
P.O. Box 5800, Albuquerque, New Mexico
Attn: Classified Document Division

Sperry Gyroscope Company
Great Neck, L.I., New York
Attn: Florence W. Turnbull, Engr. Lib.

Stanford Research Institute
Document Center
Menlo Park, California
Attn: Mary Lou Fields--Requisitions

Sylvania Electric Products, Inc.
100 First Avenue
Waltham 54, Mass.
Attn: Charles A. Thornhill, Reports Lib.
Waltham Laboratories Library

Systems Laboratories Corporation
15016 Ventura Boulevard
Sherman Oaks, California
Attn: Dr. W.C. Hoffman

Technical Research Group
17 Union Square West
New York 3, N.Y.
Attn: M.L. Henderson, Librarian

A.S. Thomas, Inc.
161 Devonshire Street
Boston 10, Mass.
Attn: A.S. Thomas, President

Westinghouse Electric Corp.
2519 Wilkens Avenue
Baltimore 3, Maryland
Attn: Kent M. Mack, Engr. Lib.

Wheeler Laboratories, Inc.
122 Cutter Mill Road
Great Neck, New York
Attn: Mr. Harold A. Wheeler

Zenith Plastics Company
Box 91, Gardena, California
Attn: Mr. S.S. Oleesky

Library, Geophysical Institute of the
University of Alaska
College, Alaska

University of California
Berkeley 4, California
Attn: S. Silver, Prof. Engr. Science
Division of Elec. Engr.
Electronics Research Lab.

Electronics Research Laboratory
University of California
332 Cory Hall, Berkeley 4, Calif.
Attn: J.R. Whinnery, Prof.

California Institute of Technology
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, California
Attn: I.E. Newlan

California Institute of Technology
1201 E. California Street
Pasadena, California
Attn: Dr. Charles H. Papas

Carnegie Institute of Technology
Schenley Park
Pittsburgh 13, Pennsylvania
Attn: Albert E. Heins

Cornell University
School of Electrical Engineering
Ithaca, New York
Attn: Prof. G.C. Dalman

University of Florida
College of Engineering
Gainesville, Florida
Attn: Prof. M.H. Latour,
Engineering Sciences Lib.

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia
Attn: Mrs. J.H. Crosland
Georgia Tech. Library

Harvard University
Technical Reports Collection
Gordon McKay Library, 303A Pierce Hall
Oxford Street, Cambridge 38, Mass.
Attn: Mrs. E.L. Hufschmidt, Lib.

Harvard College Observatory
60 Garden Street
Cambridge, Mass.
Attn: Dr. Fred L. Whipple

University of Illinois
Serials Dept. - 220S Library
Urbana, Illinois

Electrical Engineering Res. Lab.
University of Illinois
Urbana, Illinois
Attn: Antenna Lab.

The Johns Hopkins University
Homewood Campus
Department of Physics
Baltimore 18, Maryland
Attn: Professor Donald E. Kerr

The Johns Hopkins University
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Mr. George L. Seielstad,
Applied Physics Lab.

Massachusetts Institute of Technology
Research Laboratory of Electronics
Document Room 20B-221
Cambridge 39, Mass.

Massachusetts Institute of Tech.
Lincoln Laboratory
P.O. Box 73
Lexington 73, Mass.
Attn: Henry Straus, Document Rm. A-229

University of Michigan
Electronic Defense Group
Engineering Research Institute
Ann Arbor, Michigan
Attn: J.A. Boyd, Supervisor

The University of Michigan
Willow Run Laboratories
Willow Run Airport
Ypsilanti, Michigan
Attn: K.M. Siegel, Head
Theory and Analysis Dept.

University of Michigan
Willow Run Laboratories
Engineering Research Institute
Willow Run Airport
Ypsilanti, Michigan
Attn: Librarian

The University of Minnesota
Minneapolis 14, Minnesota
Attn: Mr. Robert H. Stumm, Library

Northwestern University
Microwave Laboratories
Evanston, Illinois
Attn: Professor R.E. Beam

Ohio State University
Research Foundation
Columbus 10, Ohio
Attn: Dr. T.E. Tice
Dept. of Electrical Engr.

The University of Oklahoma
Norman, Oklahoma
Attn: Prof. C.L. Farrar
Chairman Electrical Engr.

Polytechnic Institute of Brooklyn
Microwave Research Institute
55 Johnson Street
Brooklyn 1, New York
Attn: Mr. A.E. Laemmel

Syracuse University
Research Institute
Collendale D-6
Syracuse 10, New York
Attn: Dr. C.S. Grove, Jr.
Director of Engr. Research

The University of Texas
Electrical Engr. Research Lab.
P.O. Box 8026, University Station
Austin 12, Texas
Attn: J.R. Gerhardt, Assistant Dir.

The University of Texas
Defense Research Lab.
Austin 12, Texas
Attn: Claude W. Horton, Phys. Lib.

The University of Toronto
Department of Electrical Engineering
Toronto, Canada
Attn: Prof. G. Sinclair

LTI Research Foundation
P.O. Box 709
Lowell, Mass.
Attn: Dr. Charles R. Mingins

University of Washington
Dept. of Electrical Engineering
Seattle 5, Washington
Attn: G. Held, Associate Professor

Polytechnic Institute of Brooklyn
Microwave Research Institute
55 Johnson Street
Brooklyn 1, New York
Attn: Dr. A.A. Oliner

Electronics Division
Rand Corporation
1700 Main Street
Santa Monica, California
Attn: Dr. Robert Kalaba

National Bureau of Standards
Washington, D.C.
Attn: Dr. W.K. Saunders

Applied Mathematics and Statistics Lab.
Stanford University
Stanford, California
Attn: Dr. Albert H. Bowker

Dept. of Physics and Astronomy
Michigan State University
East Lansing, Michigan
Attn: Dr. A. Leitner

Ionsphere Research Laboratory
Pennsylvania State College
State College, Pennsylvania
Attn: Prof. A.H. Waynick, Director

Institute of Mathematical Sciences
25 Waverly Place
New York 3, New York
Attn: Mrs. JoAn Segal, Librarian

School of Electrical Engineering
Purdue University
Lafayette, Indiana
Attn: Professor F.V. Schultz

University of Tennessee
Knoxville, Tennessee
Attn: Dr. Fred A. Ficken

California Institute of Tech.
1201 E. California Street
Pasadena, California
Attn: Dr. A. Erdélyi

Wayne University
Detroit, Michigan
Attn: Professor A.F. Stevenson

Mathematics Department
Stanford University
Stanford, California
Attn: Dr. Harold Levine

University of Minnesota
Minneapolis 14, Minnesota
Attn: Prof. Paul C. Rosenbloom

Department of Mathematics
Stanford University
Stanford, California
Attn: Professor Bernard Epstein

Applied Physics Laboratory
The Johns Hopkins University
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Dr. B.S. Gourary

(2) Exchange and Gift Division
The Library of Congress
Washington 25, D.C.

Electrical Engineering Dept.
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
Attn: Dr. L.J. Chu

Nuclear Development Associates, Inc.
5 New Street
White Plains, New York
Attn: Library

Lebanon Valley College
Annville, Pennsylvania
Attn: Prof. B.H. Bissinger

Dept. of Physics
University of Pittsburgh
Thaw Hall
Pittsburgh 13, Pa.
Attn: Dr. Edward Gerjuoy

Dept. of Physics
Amherst College
Amherst, Massachusetts
Attn: Dr. Arnold Arons

California Institute of Tech.
Electrical Engineering
Pasadena, California
Attn: Dr. Zohrab A. Kaprielian

Dr. Rodman Doll
311 W. Cross St.
Ypsilanti, Michigan

California Institute of Technology
Pasadena 4, California
Attn: Mr. Calvin Wilcox

Mr. Robert Brockhurst
Woods Hole Oceanographic Institute
Woods Hole, Massachusetts

National Bureau of Standards
Boulder, Colorado
Attn: Dr. R. Gallet

Mrs. Jane Scanlon
224-03 67th Avenue
Bayside 64, New York

Dr. Solomon L. Schwebel
3689 Louis Road
Palo Alto, California

University of Minnesota
The University Library
Minneapolis 14, Minnesota
Attn: Exchange Division

Professor Bernard Friedman
Dept. of Mathematics
University of California
Berkeley, California

Lincoln Laboratory
Massachusetts Institute of Tech.
P.O. Box 73
Lexington 73, Massachusetts
Attn: Dr. Shou Chin Wang, Rm. C-351

Melpar, Inc.
3000 Arlington Boulevard
Falls Church, Virginia
Attn: Mr. K.S. Kelleher, Section Head

Antenna Laboratory, ERD
Hq. Air Force Cambridge Res. Center
Laurence G. Hanscom Field
Bedford, Massachusetts
Attn: Nelson A. Logan

Electronics Research Directorate
Hq. Air Force Cambridge Res. Center
Laurence G. Hanscom Field
Bedford, Massachusetts
Attn: Dr. Philip Newman, CRRK

Hq. Air Force Cambridge Res. Center
Laurence G. Hanscom Field
Bedford, Massachusetts
Attn: Mr. Francis J. Zucker

Mr. N.C. Gerson
Trapelo Road
S. Lincoln, Massachusetts

Hoffman Labs., Inc.
Advanced Development Section
3761 South Hill Street
Los Angeles 7, Calif.
Attn: Dr. Richard B. Barrar

Columbia University
Hudson Laboratories
P.O. Box 239
145 Palisade Street
Dobbs Ferry, New York
Attn: Dr. M.W. Johnson

Institute of Fluid Dynamics
and Applied Math, Univ. of Maryland
College Park, Maryland
Attn: Dr. Elliott Montroll

Dept. of Electrical Engineering
Washington University
Saint Louis 5, Missouri
Attn: Prof. J. Van Bladel

Dept. of the Navy
Office of Naval Res. Braoch Office
1030 E. Green Street
Pasadena 1, California

Braodeis University
Waltham, Massachusetta
Attn: Library

General Electric Company
Microwave Laboratory
Electronics Division
Stanford Industrial Park
Palo Alto, California
Attn: Library

Smyth Research Associates
3930 4th Avenue
San Diego 3, California
Attn: Dr. John B. Smyth

Electrical Engineering
California Inst. of Technology
Pasadena, California
Attn: Dr. Georges Weill

Naval Research Laboratory
Washington 25, D.C.
Attn: Dr. Henry J. Passerini
Code 5278 A

Dr. George Kear
5 Culver Court
Orinda, Calif.

Brooklyn Polytechnic
85 Livingston Street
Brooklyn, New York
Attn: Prof. Nathan Marcuvitz

Dept. of Electrical Engineering
Brooklyn Polytechnic
85 Livingston Street
Brooklyn, New York
Attn: Dr. Jerry Shmoya

Dept. of Mathematics
University of New Mexico
Albuquerque, New Mexico
Attn: Dr. I. Kolodner

Mathematics Dept.
Polytechnic Instituta of Brooklyn
Johnson and Jay Streets
Brooklyn, New York
Attn: Dr. Harry Hochstadt